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#### January 3, 2024



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**Stream Ciphers** 

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## Outline

## Introduction

## 2

Statistical Tests • Five Basic Tests

## 3 LFSR





## 6 Salsa20/20



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## Outline

## Introduction

2 Statistical Tests
 • Five Basic Tests

## 3 LFSF





## 5 Salsa20/20



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**Stream Ciphers** 

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## Block vs. Stream Cipher

Block Cipher

<sup>1</sup>Adding a small amount of memory to a block cipher results in a stream cipher with large blocks.

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## Block vs. Stream Cipher

#### Block Cipher

- It processes plaintext in relatively large blocks (e.g.,  $n \ge 64$  bits).
- The same function is used to encrypt successive blocks; thus (pure) block ciphers are memoryless<sup>1</sup>.



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#### Block Cipher

- It processes plaintext in relatively large blocks (e.g.,  $n \ge 64$  bits).
- The same function is used to encrypt successive blocks; thus (pure) block ciphers are memoryless<sup>1</sup>.

#### Stream Ciphers

- It processes plaintext in blocks as small as a single bit.
- The encryption function may vary as plaintext is processed.
- Thus it is said to have memory.
- It is also called state ciphers since encryption depends on not only the key and plaintext, but also on the current state.

<sup>1</sup>Adding a small amount of memory to a block cipher results in a stream cipher with lar blocks. ← □ ► ← ∂ ► ← ≥ ► ← ≥ ← ⊃ <

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#### Encryption

#### e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

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#### Encryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

	h	е	i	1	h	i	t	I	е	r
Plaintext:	001	000	010	100	001	010	111	100	000	101
Key:	111	101	110	101	111	100	000	101	110	000

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e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

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Plaintext:	001	000	010	100	001	010	111	100	000	101
Key:	111	101	110	101	111	100	000	101	110	000
Ciphertext	H: 110	0 10	100	001	110	110	111	001	110	101
	S	r	Ĩ	h	S	S	t	h	s	r

#### Decryption

#### e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

#### 



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#### Decryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

	S	r	I	h	S	s	t	h	S	r	
Ciphertext:	110	101	100	001	110	110	111	001	110	101	
Key:	111	101	110	101	111	100	000	101	110	000	
Plaintext:	001	000	010	100	001	010	111	100	000	101	
	h	е	i	1	h	i	t	I	е	r	
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#### • Provably secure ···

- Ciphertext provides no info about plaintext
- All plaintexts are equally likely
- ··· but, only when be used correctly
  - Key must be random, used only once
  - Key is known only to sender and receiver
- Note: Key is same size as message
- So, why not distribute message instead of pad?



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based on one-time pad



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#### based on one-time pad

- Except that key is relatively short
- Key is stretched into a long keystream
- Keystream is used just like a one-time pad



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#### **Main Characteristics**

- Speed: faster in hardware
- Hardware implementation cost: low
- Error propagation: limited or no error propagation
- Synchronization requirement: to allow for proper decryption, the sender and receiver must be synchronized



- A - B - N

# Difference Between Stream Cipher and Pseudorandom Generator

- The output length is not fixed and the keystream is computed recursively using an internal state and the key.
- The initial state is derived from a key and an initialization vector.
- Stream cipher is an encryption scheme based on a keystream generator.
- Encryption is defined by XORing the plaintext with the keystream



## **Classification of Stream Ciphers**

#### Synchronous Stream Ciphers:

A synchronous stream cipher is one in which the keystream is generated independently of the plaintext message and of the ciphertext.



where f is the feedback function of the cipher, g is the key stream extractor and h combines the key stream with the message stream.  $x_0$  is called the initial state and ma depend on the key.



## **Classification of Stream Ciphers**

#### • Self-Synchronous Stream Ciphers:

A self-synchronizing or asynchronous stream cipher is one in which the keystream is generated as a function of the key and a fixed number of previous ciphertext bits.





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#### Timeline

14-15 Oct 04	:	workshop hosted by ECRYPT to discuss SASC
		(The State of the Art of Stream Ciphers)
Nov 04	:	call for Primitives
29 Apr 05	:	the deadline of submission to ECRYPT.
		34 primitives have been submitted
		to ECRYPT
13 Jun 05	:	website is launched to promote the public
		evaluation of the primitives.
02-03 Feb 06	:	workshop SASC 2006 hosted by ECRYPT
Feb 06	:	The end of the first evaluation phase of
		eSTREAM.



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#### Timeline

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Jul 06	:	The beginning of the second evaluation phase of eSTREAM.
31 Jan -		
1 Feb 07	:	workshop SASC 2007 hosted
		by ECRYPT
Apr 07	:	the beginning of the third evaluation phase
		of eSTREAM
Feb 08	:	workshop SASC 2008
May 08	:	the final report of the eSTREAM
Jan 12	:	the final report of the eSTREAM
		Portfolio in 2012



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## Submission Requirements

 Submissions had to be either fast in software or resource friendly in hardware

	key	IV	tag (optional)
Profile 1 (SW )	128	64 or 128	32, 64, 96, or 128
Profile 2 (HW)	80	32 or 64	32 or 64

• Designers required to give an IP statement.



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## eSTREAM Portfolio

#### in 2008

Profile 1	Profile 2
HC-128	F-FCSR-H v2
Rabbit	Grain v1
Salsa20/12	MICKEY v2
Sosemanuk	Trivium

#### in 2012

Profile 1	Profile 2
HC-128	
Rabbit	Grain v1
Salsa20/12	MICKEY 2.0
Sosemanuk	Trivium



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	Recommendation		
Primitive	Legacy	Future	
HC-128	$\checkmark$	$\checkmark$	
Salsa20/20	$\checkmark$	$\checkmark$	
ChaCha	$\checkmark$	$\checkmark$	
SNOW 2.0	$\checkmark$	$\checkmark$	
SNOW 3G	$\checkmark$	$\checkmark$	
SOSEMANUK	$\checkmark$	$\checkmark$	



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SNOW 3G	$\checkmark$	$\checkmark$		
SOSEMANUK	$\checkmark$	$\checkmark$		
Grain	$\checkmark$	×		
Mickey 2.0	$\checkmark$	×		
Trivium	$\checkmark$	×		
Rabbit	$\checkmark$	×		



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SOSEMANUK	$\checkmark$	$\checkmark$
Grain	$\checkmark$	×
Mickey 2.0	$\checkmark$	×
Trivium	$\checkmark$	×
Rabbit	$\checkmark$	×
A5/1	×	×
A5/2	×	×
E0	×	×
RC4	×	×



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Legacy × Attack exists or security considered not sufficient. Mechanism should be replaced in Fielded products as a matter of urgency.



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- Legacy ✓ No known weaknesses at present. Better alternatives exist. Lack of security proof or limited key size.
- Future ✓ Mechanism is well studied (often with security proof). Expected to remain secure in 10-50 year lifetime.

https://www.enisa.europa.eu/publications/
algorithms-key-size-and-parameters-report-2014



- Once upon a time, not so very long ago, stream ciphers were the king of crypto
- Today, not as popular as block ciphers

• *RC*4

- Based on a changing lookup table
- Used many places (WEP ··· )
- **RFC 7465:** "Prohibiting RC4 Cipher Suites" published in Feb 2015.



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- Based on a changing lookup table
- Used many places (WEP ··· )
- **RFC 7465:** "Prohibiting RC4 Cipher Suites" published in Feb 2015.
- ChaCha20 is a modern stream cipher with good performance in s/w.
  - It has been adopted as a replacement for RC4 in several internet standards.

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Image: A math

## **RBG & PRBG**

#### Definition

A **random bit generator** is a device or algorithm which outputs a sequence of statistically independent and unbiased binary digits.



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Image: A mathematical straight and the straight and th

## **RBG & PRBG**

#### Definition

A **random bit generator** is a device or algorithm which outputs a sequence of statistically independent and unbiased binary digits.

#### Definition

A **pseudo-random bit generator (PRBG)** is a deterministic algorithm which, given a truly random binary sequence of length k, outputs a binary sequence of length  $\ell$  much larger than k which "appears" to be random. The input to the PRBG is called **seed**, while the output of the PRBG is called a **pseudo-random bit sequence**.


# PRBG & CSPRBG

#### Definition

We say that a **PRBG passes all poly-time statistical tests** if no poly-time algorithm can correctly distinguish between an output sequence of the generator and a **TRBG** of the same length with prob significantly  $> \frac{1}{2}$ .



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# PRBG & CSPRBG

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#### Definition

We say that a **PRBG passes the next-bit test** if there is no poly-time algo which, on input of the first  $\ell$  bits of an output sequence *s*, can predict the  $(\ell + 1)^{th}$  bit of *s* with prob significantly >  $\frac{1}{2}$ .



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#### Definition

A PRBG that passes the next-bit test is called a cryptographically secure PRBG.



## Linear Congruential Generator

- Designed by D. H. Lehmer in 1949
- $x_n \equiv a \cdot x_{n-1} + b \mod m$ , where  $n \ge 1$ .
- Ouput depends on the **initial seed** *x*<sub>0</sub> and *a*, *b*, & *m*.



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#### Theorem

If  $b \neq 0$ , LCG generates a sequence of length *m* iff

- $\bigcirc$  if  $p \mid m$ , then  $p \mid (a 1)$  for all prime factor p of m,
  - $\square$  if  $4 \mid m$ , then  $4 \mid (a 1)$ .

1.



# Linear Congruential Generator

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if  $p \mid m$ , then  $p \mid (a - 1)$  for all prime factor p of m,

$$\bigcirc$$
 if  $4 \mid m$ , then  $4 \mid (a - 1)$ .

,

#### LCGs are not very useful for cryptographic purpose.



### **RSA CSPRBG**

- Choose 2 large primes *p* & *q*.
- Set n = p.q
- Choose a random  $e s/t 0 < e < \phi(n) s/t gcd(e, \phi(n)) = 1$ .
- Choose a random seed  $x_0$  s/t  $1 \le x_0 \le n-1$

 $x_i \equiv x_{i-1}^e \mod n.$ 



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### **RSA CSPRBG**

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- Let *b<sub>i</sub>* be the least significant bit of *x<sub>i</sub>*.
- $\ell$  random bits are  $b_1, b_2, \ldots, b_\ell$ .

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# BBS (Blum-Blum-Shub) CSPRBG

- Generate 2 large primes  $p \& q \text{ s/t both} \equiv 3 \mod 4$
- Set *n* = *p.q*
- Select a random integer x s/t gcd(x, n) = 1
- Set initial seed  $x_0 \equiv x^2 \mod n$

 $x_i \equiv x_{i-1}^2 \mod n$ 

- Let *b<sub>i</sub>* be the least significant bit of *x<sub>i</sub>*.
- $\ell$  random bits are  $b_1, b_2, \ldots, b_\ell$ .



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#### Outline

#### 1 Introduction



# Statistical Tests

Five Basic Tests







#### Salsa20/20



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**Stream Ciphers** 

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Let s = s<sub>0</sub>, s<sub>1</sub>, s<sub>2</sub>,... be an infinite sequence. The subsequence consisting of the first *n* terms of *s* is denoted by s<sup>n</sup> = s<sub>0</sub>, s<sub>1</sub>,..., s<sub>n-1</sub>.



- Let  $s = s_0, s_1, s_2, ...$  be an infinite sequence. The subsequence consisting of the first *n* terms of *s* is denoted by  $s^n = s_0, s_1, ..., s_{n-1}$ .
- A run of s is a subsequence of s consisting of consecutive 0's or consecutive 1's which is neither preceded nor succeeded by the same symbol. A run of 0's is called a gap, while a run of 1's is called a block.



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#### Definition

Let  $s = s_0, s_1, s_2, ...$  be a periodic sequence of period *N*. The autocorrelation function of *s* is the integer-valued function *C*(*t*) defined as

$$C(t) = \frac{1}{N} \sum_{i=0}^{N-1} (2.s_i - 1).(2s_{i+t} - 1), \quad for \ 0 \le t \le N-1.$$



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*C*(*t*) measures the amount of similarity between the sequence *s* and a shift of *s* by *t* positions,  $|t|_{x}$  is a random periodic sequence of period *N*, then |N.C(t)| can be expected to be quite small for all values of *t*, 0 < t < N.

Let s be a periodic sequence of period N. Golomb's randomness postulates are the following:

(1)

In the cycle  $s^N$  of s, the number of 1's differs from the number of 0's by at most 1.



Image: A math

Let s be a periodic sequence of period N. **Golomb's randomness postulates** are the following:

- In the cycle  $s^{N}$  of s, the number of 1's differs from the number of 0's by at most 1.
- In the cycle s<sup>N</sup>, at least half the runs have length 1, at least one-fourth have length 2, at least one-eighth have length 3, etc., as long as the number of runs so indicated exceeds 1. Moreover, for each of these lengths, there are (almost) equally many gaps and blocks.



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$$N \times C(t) = \sum_{i=0}^{N-1} (2.s_i - 1) \cdot (2s_{i+t} - 1) = \begin{cases} N, & \text{if } t = 0, \\ K, & \text{if } 1 \le t \le N - 1. \end{cases}$$



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A binary sequence which satisfies Golomb's randomness postulates is called a pseudo-noise sequence or a pn-sequence.



### Frequency Test (Monobit Test)

- The purpose of this test is to determine whether the number of 0's and 1's in *s* are approximately the same, as would be expected for a random sequence.
- Let  $s = s_0, s_1, s_2, \ldots, s_{n-1}$  be a binary sequence of length *n*.
- Let  $n_0, n_1$  denote the number of 0's and 1's in *s*, respectively.



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- Let  $n_0, n_1$  denote the number of 0's and 1's in s, respectively.
- The statistic used is

$$X_1 = \frac{(n_0 - n_1)^2}{n}$$

which approximately follows a  $\chi^2$  distribution with 1 degree of freedom if  $n \ge 10$ .



## Serial Test (2-bit Test)

• The purpose of this test is to determine whether the number of occurrences of 00, 01, 10, and 11 as subsequences of *s* are approximately the same, as would be expected for a random sequence.



 ${}^{2}n_{00} + n_{01} + n_{10} + n_{11} = (n-1)$  since the subsequences are allowed to overlap.  $\exists \neg \land \land \land$ Dhananjoy Dey (Indian Institute of Informa Stream Ciphers January 3, 2024 31/77

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- Let n<sub>0</sub>, n<sub>1</sub> denote the number of 0's and 1's in s, respectively, and let n<sub>00</sub>, n<sub>01</sub>, n<sub>10</sub>, n<sub>11</sub> denote the number of occurrences of 00, 01, 10, 11 in s, respectively<sup>2</sup>.



 $\frac{2n_{00} + n_{01} + n_{10} + n_{11}}{2n_{00} + n_{01} + n_{10} + n_{11}} = (n - 1) \text{ since the subsequences are allowed to overlap.} = 9 < 0$ Dhananjoy Dey (Indian Institute of Informa Stream Ciphers January 3, 2024 31/77

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- The statistic used is

$$X_2 = \frac{4}{n-1}(n_{00}^2 + n_{01}^2 + n_{10}^2 + n_{11}^2) - \frac{2}{n}(n_0^2 + n_1^2) + 1$$

which approximately follows a  $\chi^2$  distribution with 2 degrees of freedom if  $n \ge 21$ .



 $n_{00}^2 + n_{01} + n_{10} + n_{11} = (n-1)$  since the subsequences are allowed to overlap. Dhananjoy Dey (Indian Institute of Informa Stream Ciphers January 3, 2024 31/77

#### Poker test

- Let *m* be a positive integer such that  $\lfloor \frac{n}{m} \rfloor \ge 5.2^m$ , and let  $k = \lfloor \frac{n}{m} \rfloor$ .
- Divide the sequence *s* into *k* non-overlapping parts each of length *m*
- Let  $n_i$  be the number of occurrences of the  $i^{th}$  type of sequence of length m,  $1 \le i \le 2^m$ .



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#### **Five Basic Tests**

#### Poker test

- Let *m* be a positive integer such that  $\lfloor \frac{n}{m} \rfloor \ge 5.2^m$ , and let  $k = \lfloor \frac{n}{m} \rfloor$ .
- Divide the sequence s into k non-overlapping parts each of length m
- Let  $n_i$  be the number of occurrences of the  $i^{th}$  type of sequence of length m,  $1 \le i \le 2^m$ .
- The poker test<sup>3</sup> determines whether the sequences of length *m* each appear approximately the same number of times in *s*, as would be expected for a random sequence.
- The statistic used is

$$X_3 = \frac{2^m}{k} \left( \sum_{i=1}^{2^m} n_i^2 \right) - k$$

which approximately follows a  $\chi^2$  distribution with  $2^m - 1$  degrees of freedom.

<sup>3</sup>Note that the poker test is a generalization of the frequency test: setting m = 1 in the poker test yields the frequency test.

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#### **Five Basic Tests**

#### Runs test

The purpose of the runs test is to determine whether the number of runs of ۰ various lengths in the sequence *s* is as expected for a random sequence.



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#### **Five Basic Tests**

#### Runs test

- The purpose of the runs test is to determine whether the number of runs of ٠ various lengths in the sequence s is as expected for a random sequence.
- The expected number of gaps (or blocks) of length *i* in a random sequence of ۰ length *n* is  $e_i = (n - i + 3)/2^{i+2}$ .
- Let k be equal to the largest integer i for which  $e_i \ge 5$ .
- Let  $B_i, G_i$  be the number of blocks and gaps, respectively, of length i in s for each  $i, 1 \leq i \leq k$ .
- The statistic used is

$$X_4 = \sum_{i}^{k} \frac{(B_i - e_i)^2}{e_i} + \sum_{i}^{k} \frac{(G_i - e_i)^2}{e_i}$$

which approximately follows a  $\chi^2$  distribution with 2k-2 degrees of freedom.



### Autocorrelation test

- The purpose of this test is to check for correlations between the sequence *s* and (non-cyclic) shifted versions of it.
- Let *d* be a fixed integer,  $1 \le d \le \lfloor n/2 \rfloor$ .
- The number of bits in *s* not equal to their *d*-shifts is  $A(d) = \sum_{i=0}^{n-d-1} s_i \oplus s_{i+d}$ .
- The statistic used is

$$X_5 = \frac{2(A(d) - \frac{n-d}{2})}{\sqrt{n-d}}$$

which approximately follows an N(0, 1) distribution if  $n - d \ge 10$ . Since small values of A(d) are as unexpected as large values of A(d), a two-sided test should be used.



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#### Outline



Statistical TestsFive Basic Tests







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## Linear Feedback Shift Registers (LFSR)

 A standard way of producing a binary stream of data is to use a feedback shift register.



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### Linear Feedback Shift Registers (LFSR)

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- These are small circuits containing a number of memory cells, each of which holds one bit of information.
- The set of such cells forms a register.
- In each cycle a certain predefined set of cells are '**tapped**' and their value is passed through a function, called the feedback function.



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- In each cycle a certain predefined set of cells are 'tapped' and their value is passed through a function, called the feedback function.
- The register is then shifted down by one bit, with the output bit of the feedback shift register being the bit that is shifted out of the register.
- The combination of the tapped bits is then fed into the empty cell at the top of the register.



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#### Linear Feedback Shift Registers (LFSR)



Figure: LFSR of length L



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### Linear Feedback Shift Registers (LFSR)



Figure: LFSR of length L

• This LFSR is denoted by  $\langle L, C(D) \rangle$ , where

 $C(D) = 1 + c_1 D + c_2 D^2 + \dots + c_L D^L \in GF(2)[D]$ 



is the connection polynomial.

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# Linear Feedback Shift Registers (LFSR)

#### Definition

- A LFSR of degree L (or length L) is defined by feedback coefficients c<sub>1</sub>,..., c<sub>L</sub> ∈ GF(2).
- The initial state is an *L*-bit word *S* = (*s*<sub>*L*-1</sub>,...,*s*<sub>1</sub>,*s*<sub>0</sub>) and new bits are generated by the recursion

$$s_j = (c_1.s_{j-1} \oplus c_2s_{j-2} \oplus \ldots \oplus c_L.s_{j-L}), \text{ for } j \ge L$$

- At each iteration step, the state S is updated from (s<sub>j-1</sub>,..., s<sub>j-L</sub>) to (s<sub>j</sub>, s<sub>j-1</sub>,..., s<sub>j-L+1</sub>), by shifting the register to the right. The rightmost bit s<sub>j-L</sub> is output.
- The output of an LFSR is called a linear recurring sequence.

# Linear Feedback Shift Registers (LFSR)



- Let the length of the register be *L*.
- One defines a set of bits (c<sub>1</sub>,..., c<sub>L</sub>) where c<sub>i</sub> = 1 if that cell is tapped and c<sub>i</sub> = 0 otherwise.
- The initial internal state of the register is given by the bit sequence  $(s_{L-1}, \ldots, s_1, s_0)$ .
- The output sequence is then defined to be

 $s_0, s_1, s_2, \ldots, s_{L-1}, s_L, s_{L+1}, \ldots$  where for  $j \ge L$  we have

$$s_j = c_1 \cdot s_{j-1} \oplus c_2 s_{j-2} \oplus \ldots \oplus c_L \cdot s_{j-L}.$$



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Image: A math
# Linear Feedback Shift Registers (LFSR)

### Example (LFSR)





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# Linear Feedback Shift Registers (LFSR)



### • Connection polynomial:



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# Linear Feedback Shift Registers (LFSR)



- Connection polynomial:  $c(x) = x^4 + x + 1$
- Initial state is (1, 1, 0, 1)



# Linear Feedback Shift Registers (LFSR)

#### Example (LFSR) 1101 $\rightarrow$ 1 $0110 \rightarrow 0$ $0011 \rightarrow 1$ $1001 \rightarrow 1$ 0100 $\rightarrow 0$ $0010 \rightarrow 0$ $0001 \rightarrow 1$ 1000 $\rightarrow 0$ 1100 $\rightarrow 0$ 1110 $\rightarrow 0$ $1111 \rightarrow 1$ $0111 \rightarrow 1$ $1011 \rightarrow 1$

 $0101 \rightarrow 1$ 1010

 $\rightarrow$  1

# Linear Feedback Shift Registers (LFSR)

### Example (LFSR)

1101	$\rightarrow$	1
0110	$\rightarrow$	0
0011	$\rightarrow$	1
1001	$\rightarrow$	1
0100	$\rightarrow$	0
0010	$\rightarrow$	0
0001	$\rightarrow$	1
1000	$\rightarrow$	0
1100	$\rightarrow$	0
1110	$\rightarrow$	0
1111	$\rightarrow$	1
0111	$\rightarrow$	1
1011	$\rightarrow$	1
0101	$\rightarrow$	1
1010	$\rightarrow$	1
1101		

# Linear Feedback Shift Registers (LFSR)

#### Definition

Let  $s_0, s_1, s_2, ...$  be a linear recurring sequence. The period of the sequence is the smallest integer  $N \ge 1$  s/t

 $s_{j+N} = s_j$ 

for all sufficiently large values of j.



# Linear Feedback Shift Registers (LFSR)

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 $s_{j+N} = s_j$ 

for all sufficiently large values of j.

#### Proposition

The period of a sequence generated by an LFSR of degree *n* is at most  $2^n - 1$ .



## Linear Complexity

#### Definition

The **linear complexity** of an infinite binary sequence s, denoted L(s), is defined as follows:

- If s is the zero sequence  $s = 0, 0, 0, \dots$ , then L(s) = 0;
- $\bigcirc$  if no LFSR generates s, then  $L(s) = \infty$ ;
- 0 otherwise, L(s) is the length of the shortest LFSR that generates s.



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  - $\square$  otherwise, L(s) is the length of the shortest LFSR that generates s.

### Definition

The linear complexity of a finite binary sequence  $s^n$ , denoted  $L(s^n)$ , is the length of the shortest LFSR that generates a sequence having  $s^n$ as its first *n* terms.



# Properties of Linear Complexity

- **(**) For any  $n \ge 1$ , the linear complexity of the subsequence  $s^n$  satisfies  $0 \le L(s^n) \le n$ .
- $L(s^n) = 0 iff s^n is the zero sequence of length n.$
- $L(s^n) = n \text{ iff } s^n = 0, 0, 0, \dots, 0, 1.$
- If s is periodic with period N, then  $L(s) \leq N$ .
- $U(s \oplus t) \le L(s) + L(t), \text{ where } s \oplus t \text{ denotes the bitwise XOR of } s \text{ and } t.$



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# Non-linear FSR (NLFSR)

#### Example

### • Consider a 4-stage NFSR with a feedback function

 $f(x_0, x_1, x_2, x_3) = 1 + x_0 + x_1 + x_1 x_2 x_3$ 

# Non-linear FSR (NLFSR)

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# Non-linear FSR (NLFSR)

Example

 $f(x_0, x_1, x_2, x_3) = 1 + x_0 + x_1 + x_1 x_2 x_3 - de Bruijn FSR$ 

0001	$\rightarrow$	- 1
0000	$\rightarrow$	0
1000	$\rightarrow$	0
1100	$\rightarrow$	0
1110	$\rightarrow$	0
1111	$\rightarrow$	1
0111	$\rightarrow$	1
1011	$\rightarrow$	1
1101	$\rightarrow$	1
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0011	$\rightarrow$	1
1001	$\rightarrow$	1
0100	$\rightarrow$	0
1010	$\rightarrow$	0
0101	$\rightarrow$	1
0010	$\rightarrow$	1

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## Stream Ciphers Based on LFSRs

- Combination generator
- Filter generator
- Shrinking generator



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## Non-linear Combination Generator

- One general technique for destroying the linearity inherent in LFSRs is to use several LFSRs in parallel.
- The key stream is generated as a non-linear function *f* of the outputs of the component LFSRs.
- Such key stream generators are called non-linear combination generators, and *f* is called the combining function.



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### Example (Geffe Generator)



# Non-linear Combination Generator

### Example (Geffe Generator)



 Consider 3 maximum-length LFSRs whose lengths L<sub>1</sub>, L<sub>2</sub>, L<sub>3</sub> are pairwise relatively prime, with nonlinear combining function

 $f(x_1, x_2, x_3) = x_1 x_2 \oplus (1 + x_2) x_3 = x_1 x_2 \oplus x_2 x_3 \oplus x_3.$ 

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### Example (Geffe Generator)



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 $f(x_1, x_2, x_3) = x_1 x_2 \oplus (1 + x_2) x_3 = x_1 x_2 \oplus x_2 x_3 \oplus x_3.$ 

• The keystream generated has period  $(2^{L_1} - 1)(2^{L_2} - 1)(2^{L_3} - 1)$  and linear complexity  $L = L_1L_2 + L_2L_3 + L_3$ .

## **Filter Generator**

- A filter generator is a running-key generator for stream cipher applications.
- It consists of a single LFSR which is filtered by a non-linear function *f*.



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- A control LFSR R<sub>1</sub> is used to select a portion of the output sequence of a second LFSR R<sub>2</sub>
- Due to its simplicity, it was a promising candidate for high-speed encryption applications.



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## Outline



2 Statistical Tests• Five Basic Tests

### 3 LFSF





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• A self-modifying lookup table (or Synchronous stream cipher) designed by Ron Rivest in 1987.

RC4

- Table always contains a permutation of the byte values 0, 1, ..., 255
- Initialize the permutation using key
- At each step, RC4 does the following



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  - Efficient in software
- Each step of A5/1 produces only a bit



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- Each step of A5/1 produces only a bit
  - Efficient in hardware



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# RC4 Key Scheduling Algorithm (KSA)

**Input**: Key array K[0], K[1], ..., K[n-1] of *n* bytes,  $1 \le n \le 255$ **Output**: State array S[0], S[1], ..., S[255]

RC4

- 1: **for** i = 0 to 255 **do**
- 2: S[i] = i
- 3: end for
- 4: j = 0
- 5: **for** i = 0 to 255 **do**
- 6:  $j = (j + S[i] + K[i \mod n]) \mod 256$
- 7: Swap the values of S[i] and S[j]
- 8: end for



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# RC4 Pseudorandom Generation Algorithm (PRGA)

• For each keystream byte, swap elements in table and select byte

Input: State array S[0], S[1], ..., S[255]Output: Output bytes B1: i = 02: j = 03: while Keystream is generated do 4: i = i + 15:  $j = (j + S[i]) \mod 256$ 6: Swap the values of S[i] and S[j]7:  $B = S[(S[i] + S[j]) \mod 256]$ 8: Output B9: end while

- Use keystream bytes like a one-time pad
- Note: first 256 bytes should be discarded
  - Otherwise, related key attack exists



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## Outline



2 Statistical Tests
 • Five Basic Tests

### 3 LFSF









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### Trivium

- Designed by De Canniére and Preneel in 2006 as part of eSTREAM competition
- Intended to be simple and efficient (especially in hardware)



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## **Trivium Hardware**





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## Trivium

#### • Parameters:

Key size: 80 bit, IV size: 80 bit, Internal state: 288 bit



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## Trivium

#### Parameters:

Key size: 80 bit, IV size: 80 bit, Internal state: 288 bit

• Three coupled FSR of degree 93, 84, and 111.

### Initialization:

- 80-bit key in left-most registers of first FSR
- 80-bit IV in left-most registers of second FSR
- Remaining registers set to 0, except for three right-most (all 1s) registers of third FSR
- run for  $4 \times 288$  clock ticks to finish initialization

https://www.ecrypt.eu.org/stream/p3ciphers/trivium/trivium\_p3.pdf


## **Trivium-Initialization**

For i = 1 to  $4 \times 288$  do

- $2 t_2 \leftarrow s_{162} + s_{175}s_{176} + s_{177} + s_{264}$



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Image: A math

## **Trivium-Initialization**

- For i = 1 to  $4 \times 288$  do

  - $2 t_2 \leftarrow s_{162} + s_{175}s_{176} + s_{177} + s_{264}$

  - $(s_1, s_2, \dots, s_{93}) \leftarrow (t_3, s_1, \dots, s_{92})$
  - **5**  $(s_{94}, s_{95}, \dots, s_{177}) \leftarrow (t_1, s_{94}, \dots, s_{176})$
  - $(s_{178}, s_{279}, \dots, s_{288}) \leftarrow (t_2, s_{178}, \dots, s_{287})$



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# **Trivium-Initialization**

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  - $(s_{178}, s_{279}, \dots, s_{288}) \leftarrow (t_2, s_{178}, \dots, s_{287})$

Note: no random bits output. This is just initialization.



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#### Trivium

# **Trivium-Iteration**

- For i = 1 to  $N(\le 2^{64})$  do
  - $\bullet t_1 \leftarrow s_{66} + s_{93}$
  - **2**  $t_2 \leftarrow s_{162} + s_{177}$
  - $I_3 \leftarrow s_{243} + s_{288}$



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#### Trivium

### **Trivium-Iteration**

- For i = 1 to  $N(\le 2^{64})$  do
  - $\bullet t_1 \leftarrow s_{66} + s_{93}$
  - $t_2 \leftarrow s_{162} + s_{177}$
  - **③**  $t_3 \leftarrow s_{243} + s_{288}$
  - $z_i \leftarrow t_1 + t_2 + t_3 1 bit of key stream$
  - **5**  $t_1 \leftarrow t_1 + s_{91}s_{92} + s_{171}$

  - $t_3 \leftarrow t_3 + s_{286} s_{287} + s_{69}$
  - **8** ( $s_1, s_2, ..., s_{93}$ ) ← ( $t_3, s_1, ..., s_{92}$ )
  - **9** (*s*<sub>94</sub>, *s*<sub>95</sub>, . . . , *s*<sub>177</sub>) ← ( $t_1$ , *s*<sub>94</sub>, ←, *s*<sub>176</sub>)
  - $(s_{178}, s_{279}, \dots, s_{288}) \leftarrow (t_2, s_{178}, \dots, s_{287})$



# Outline



2 Statistical Tests
• Five Basic Tests

#### 3 LFSF









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• Designed by Daniel J. Bernstein in 2005



<sup>4</sup>Strings are interpreted in little-endian notation

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- Designed by Daniel J. Bernstein in 2005
- It is based on three simple operations:
  - modular addition of 32-bit words a and  $b \mod 2^{32}$ , denoted by  $a \boxplus b$ ,
  - XOR-sum of 32-bit words a and b, denoted by  $a \oplus b$ ,
  - circular left shift of a 32-bit word a by t positions, denoted by  $a \ll t$ .



<sup>4</sup>Strings are interpreted in little-endian notation Dhananjoy Dev (Indian Institute of Informa Stream Ciphers

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- Designed by Daniel J. Bernstein in 2005 •
- It is based on three simple operations:
  - modular addition of 32-bit words a and b mod  $2^{32}$ , denoted by  $a \equiv b$ ,
  - XOR-sum of 32-bit words a and b, denoted by  $a \oplus b$ ,
  - circular left shift of a 32-bit word a by t positions, denoted by  $a \ll t$ .
- The Salsa20/20 cipher takes a 256-bit key, a 64-bit nonce and a 64-bit counter.
- The state array S of Salsa20 is a 4 × 4 matrix of sixteen 32-bit words<sup>4</sup>



<sup>4</sup>Strings are interpreted in little-endian notation Dhananiov Dev (Indian Institute of Informa Stream Ciphers

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The state array S:

$$S = \begin{pmatrix} y_0 & y_1 & y_2 & y_3 \\ y_4 & y_5 & y_6 & y_7 \\ y_8 & y_9 & y_{10} & y_{11} \\ y_{12} & y_{13} & y_{14} & y_{15} \end{pmatrix}$$



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The state array S:

$$S = \begin{pmatrix} y_0 & y_1 & y_2 & y_3 \\ y_4 & y_5 & y_6 & y_7 \\ y_8 & y_9 & y_{10} & y_{11} \\ y_{12} & y_{13} & y_{14} & y_{15} \end{pmatrix}$$

- Salsa20 is based on quarter-rounds, row-rounds and columnrounds.
- The quarter-rounds operate on four words, the row-rounds transform the four rows and the column-rounds transform the fourcolumns of the state matrix.



# Salsa20/20: Quarter-round





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# Salsa20/20: Row-round

$$row - round(S) = \begin{pmatrix} z_0 & z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 & z_7 \\ z_8 & z_9 & z_{10} & z_{11} \\ z_{12} & z_{13} & z_{14} & z_{15} \end{pmatrix},$$

where

 $(z_0, z_1, z_2, z_3) = quarter-round(y_0, y_1, y_2, y_3),$  $(z_5, z_6, z_7, z_4) = quarter-round(y_5, y_6, y_7, y_4),$  $(z_{10}, z_{11}, z_8, z_9) = quarter-round(y_{10}, y_{11}, y_8, y_9),$  $(z_{15}, z_{12}, z_{13}, z_{14}) = quarter-round(y_{15}, y_{12}, y_{13}, y_{14}).$ 



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# Salsa20/20: Column-round

- The *column-round* function is the transpose of the row-round function: the words in the columns are permuted, the quarter-round map is applied to each of the columns and the permutation is reversed.
- Let *S* be a state matrix as above; then

 $\operatorname{column-round}(S) = (\operatorname{row-round}(S^T))^T$ .



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# Salsa20/20: Double-round

- A *double-round* is the composition of a column-round and a row-round.
- Let *S* be a state matrix as above; then

double-round(*S*) = row-round(column-round(*S*)).



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# Salsa20/20: Double-round

- A *double-round* is the composition of a column-round and a row-round.
- Let *S* be a state matrix as above; then

double-round(*S*) = row-round(column-round(*S*)).

Salsa20 runs 10 successive double-rounds, i.e., 20 quarter-rounds, in order to generate 64 bytes of output.

The *initial state* depends on the *key*, a *nonce* and a *counter*.



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#### • The Salsa20/20 stream cipher takes a 256-bit key



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- The Salsa20/20 stream cipher takes a 256-bit key  $k = (k_1, ..., k_8)$  and a unique 64-bit message number  $n = (n_1, n_2)$  (nonce) as input.
- A 64-bit block counter  $b = (b_1, b_2)$  is initially set to zero.
- The initialization algorithm copies *k*, *n*, *b* and the four 32-bit constants

 $y_0 = 61707865, y_5 = 3320646E, y_{10} = 79622D32, \& y_{15} = 6B206574$ 

into the sixteen 32-bit words of the Salsa20 state matrix:



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The state array S:

$$S = \begin{pmatrix} y_0 & k_1 & k_2 & k_3 \\ k_4 & y_5 & n_1 & n_2 \\ b_1 & b_2 & y_{10} & k_5 \\ k_6 & k_7 & k_8 & y_{15} \end{pmatrix}$$

• The key stream generator computes the output state by *10 double-round* iterations and a final addition mod 2<sup>32</sup> of the initial state matrix:

 $Salsa20_k(n, b) = S + double-round^{10}(S).$ 



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The state array S:

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ChaCha20 is a modification of Salsa20 Carbon ChaCha20 Is a modification of Salsa20

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# ChaCha20

- ChaCha20 is a stream cipher intended to be extremely efficient in s/w, introduced in 2008.
- It is not an eSTREAM candidate!



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# ChaCha20

- ChaCha20 is a stream cipher intended to be extremely efficient in s/w, introduced in 2008.
- It is not an eSTREAM candidate! "Post-eSTREAM cryptography"
- It is available as a replacement for RC4 in many systems.
- It is combined with the Poly1305 message authentication code to construct an authenticated encryption (AE) scheme widely used in the TLS protocol.



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# ChaCha20 Quarter-round

- Let y = (a, b, c, d) be a sequence of four 32-bit words.
- Then a ChaCha quarter-round updates (*a*, *b*, *c*, *d*) as follows:

$$\begin{array}{l} \textcircled{0} \quad a \leftarrow a + b; \quad d \leftarrow d \oplus a; \quad d \ll 16; \\ \textcircled{0} \quad c \leftarrow c + d; \quad b \leftarrow b \oplus c; \quad b \ll 12; \\ \hline \textcircled{0} \quad a \leftarrow a + b; \quad d \leftarrow d \oplus a; \quad d \ll 8; \\ \hline \textcircled{0} \quad c \leftarrow c + d; \quad b \leftarrow b \oplus c; \quad b \ll 7; \end{array}$$



# ChaCha20 Double-round

- ChaCha20 also runs 10 double-rounds.
- However, a ChaCha double-round consists of a column-round and a diagonal-round
- A ChaCha double-round is defined by the 8 ChaCha quarter-rounds

column-round	quarter-round( $y_0, y_4, y_8, y_{12}$ ) quarter-round( $y_1, y_5, y_9, y_{13}$ ) quarter-round( $y_2, y_6, y_{10}, y_{14}$ )) quarter-round( $y_3, y_7, y_{11}, y_{15}$ )
diagonal-round	quarter-round $(y_0, y_5, y_{10}, y_{15})$ quarter-round $(y_1, y_6, y_{11}, y_{12})$ quarter-round $(y_2, y_7, y_8, y_{13})$ quarter-round $(y_3, y_4, y_9, y_{14})$



## ChaCha20

The state array S:

$$S = \begin{pmatrix} y_0 & y_1 & y_2 & y_3 \\ k_1 & k_2 & k_3 & k_4 \\ k_5 & k_6 & k_7 & k_8 \\ b & n_1 & n_2 & n_3 \end{pmatrix}$$

- The ChaCha20 stream cipher takes a 256-bit key  $k = (k_1, ..., k_8)$  and a unique 96-bit message number  $n = (n_1, n_2, n_3)$  (nonce) as input.
- A 32-bit block counter b is initially set to zero and the four 32-bit constants

 $y_0 = 61707865, y_1 = 3320646E, y_2 = 79622D32, y_3 = 6B206574$ 

$$ChaCha_k(n, b) = S + double-round^{10}(S).$$



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#### • Stream ciphers were popular in the past



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# **Stream Ciphers**

#### Stream ciphers were popular in the past

- Efficient in hardware
- Speed was needed to keep up with voice, etc.
- Today, processors are fast, so software-based crypto is usually more than fast enough



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# **Stream Ciphers**

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- Future of stream ciphers?



# **Stream Ciphers**

#### Stream ciphers were popular in the past

- Efficient in hardware
- Speed was needed to keep up with voice, etc.
- Today, processors are fast, so software-based crypto is usually more than fast enough
- Future of stream ciphers?
  - Shamir declared "the death of stream ciphers"
  - May be greatly exaggerated ...



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#### Thank you very much for your attention!



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