Stream Ciphers

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1

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Outline

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Block vs. Stream Cipher

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Block vs. Stream Cipher

- **Block Cipher**
  - It processes plaintext in relatively large blocks (e.g., $n \geq 64$ bits).
  - The same function is used to encrypt successive blocks; thus (pure) block ciphers are memoryless[1].

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Block vs. Stream Cipher

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  - The same function is used to encrypt successive blocks; thus (pure) block ciphers are memoryless[1].
- **Stream Ciphers**
  - It processes plaintext in blocks as small as a single bit.
  - The encryption function may vary as plaintext is processed.
  - Thus it is said to have memory.
  - It is also called state ciphers since encryption depends on not only the *key* and *plaintext*, but also on the *current state*.

[1] Adding a small amount of memory to a block cipher results in a stream cipher with large blocks.

One-Time Pad

Encryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

Encryption: Plaintext $\oplus$ Key = Ciphertext

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Encryption: Plaintext $\oplus$ Key = Ciphertext

	h	e	i	l	h	i	t	l	e	r
Plaintext:	001	000	010	100	001	010	111	100	000	101
Key:	111	101	110	101	111	100	000	101	110	000

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e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

Encryption: Plaintext $\oplus$ Key = Ciphertext

	h	e	i	l	h	i	t	l	e	r
Plaintext:	001	000	010	100	001	010	111	100	000	101
Key:	111	101	110	101	111	100	000	101	110	000
Ciphertext:	110	101	100	001	110	110	111	001	110	101
	s	r	l	h	s	s	t	h	s	r

One-Time Pad

Decryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

Decryption: Ciphertext $\oplus$ Key = Plaintext

One-Time Pad

Decryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

Decryption: Ciphertext $\oplus$ Key = Plaintext

	s	r	l	h	s	s	t	h	s	r
Ciphertext:	110	101	100	001	110	110	111	001	110	101
Key:	111	101	110	101	111	100	000	101	110	000
Plaintext:	001	000	010	100	001	010	111	100	000	101
	h	e	i	l	h	i	t	l	e	r

One-Time Pad

- **Provably secure** $\cdots$
  - Ciphertext provides no info about plaintext
  - All plaintexts are equally likely
- $\cdots$ but, only when be used correctly
  - Key must be random, used only once
  - Key is known only to sender and receiver
- Note: Key is same size as message
- So, why not distribute message instead of pad?

Stream Cipher

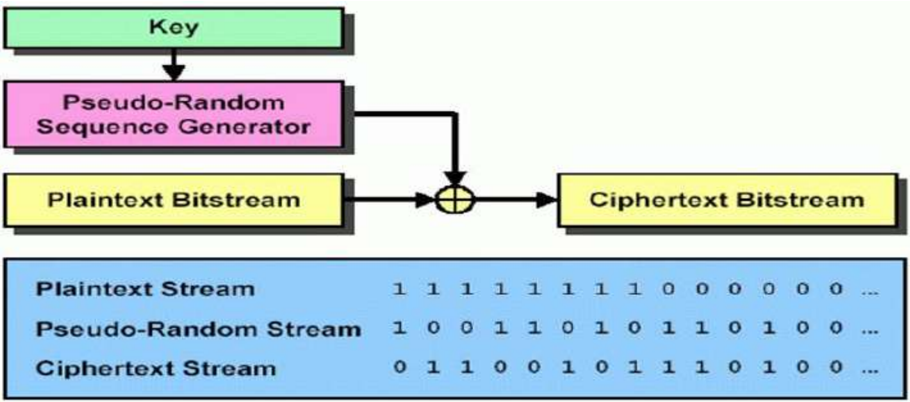
based on one-time pad

Stream Cipher

based on one-time pad

- Except that key is relatively short
- Key is stretched into a long keystream
- Keystream is used just like a one-time pad

Stream Cipher



Stream Cipher

Main Characteristics

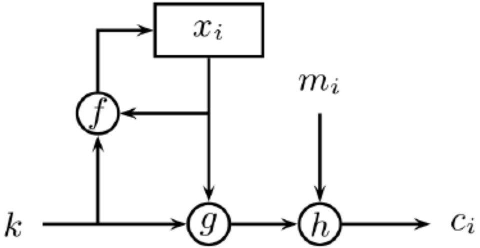
- **Speed:** faster in hardware
- **Hardware implementation cost:** low
- **Error propagation:** limited or no error propagation
- **Synchronization requirement:** to allow for proper decryption, the sender and receiver must be synchronized

Difference Between Stream Cipher and Pseudorandom Generator

- The output length is not fixed and the keystream is computed recursively using an internal state and the key.
- The initial state is derived from a key and an initialization vector.
- Stream cipher is an encryption scheme based on a keystream generator.
- Encryption is defined by XORing the plaintext with the keystream

Classification of Stream Ciphers

- **Synchronous Stream Ciphers:**
  A synchronous stream cipher is one in which the keystream is generated independently of the plaintext message and of the ciphertext.

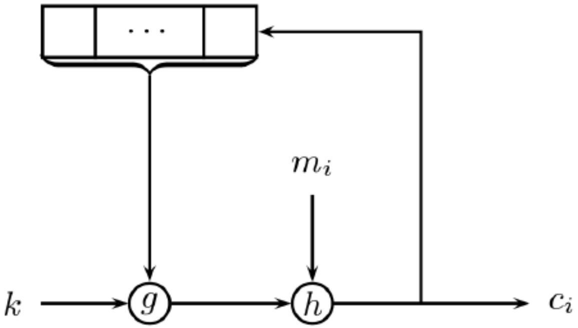


where f is the feedback function of the cipher, g is the key stream extractor and h combines the key stream with the message stream. x_0 is called the initial state and may depend on the key.

Classification of Stream Ciphers

- **Self-Synchronous Stream Ciphers:**

  A self-synchronizing or asynchronous stream cipher is one in which the keystream is generated as a function of the key and a fixed number of previous ciphertext bits.



The eSTREAM Project

Timeline

14-15 Oct 04	:	workshop hosted by ECRYPT to discuss SASC (The State of the Art of Stream Ciphers)
Nov 04	:	call for Primitives
29 Apr 05	:	the deadline of submission to ECRYPT. 34 (32 + 2) primitives have been submitted to ECRYPT
13 Jun 05	:	website is launched to promote the public evaluation of the primitives.
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Feb 06	:	The end of the first evaluation phase of eSTREAM.

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https://www.ecrypt.eu.org/stream/

The eSTREAM Project

Timeline

Jul 06	:	The beginning of the second evaluation phase of eSTREAM.
31 Jan - 01 Feb 07	:	workshop SASC 2007 hosted by ECRYPT
Apr 07	:	the beginning of the third evaluation phase of eSTREAM
Feb 08	:	workshop SASC 2008
May 08	:	the final report of the eSTREAM
Jan 12	:	the final report of the eSTREAM Portfolio in 2012

Submission Requirements

- Submissions had to be either fast in software or resource friendly in hardware

	key	IV	tag (optional)
Profile 1 (SW)	128	64 or 128	32, 64, 96, or 128
Profile 2 (HW)	80	32 or 64	32 or 64

- Designers required to give an IP statement.

eSTREAM Portfolio

in 2008

Profile 1	Profile 2
HC-128	F-FCSR-H v2
Rabbit	Grain v1
Salsa20/12	MICKEY v2
Sosemanuk	Trivium

in 2012

Profile 1	Profile 2
HC-128	
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Recommended Stream Ciphers (ENISA - Nov 2014)

Primitive	Recommendation: Legacy	Recommendation: Future
HC-128	✓	✓
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Trivium	✓	×
Rabbit	✓	×
A5/1	×	×
A5/2	×	×
E0	×	×
RC4	×	×

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Legacy ✓	No known weaknesses at present. Better alternatives exist. Lack of security proof or limited key size.

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Legacy × Attack exists or security considered not sufficient. Mechanism should be replaced in Fielded products as a matter of urgency.

Legacy ✓ No known weaknesses at present. Better alternatives exist. Lack of security proof or limited key size.

Future ✓ Mechanism is well studied (often with security proof). Expected to remain secure in 10-50 year lifetime.

https://www.enisa.europa.eu/publications/algorithms-key-size-and-parameters-report-2014

Stream Ciphers

- Once upon a time, not so very long ago, stream ciphers were the king of crypto
- Today, not as popular as block ciphers
- *RC4*
  - Based on a changing lookup table
  - Used many places (WEP $\cdots$)
- **RFC 7465:** “Prohibiting RC4 Cipher Suites" published in Feb 2015.

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- $RC4$
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- **RFC 7465:** “Prohibiting RC4 Cipher Suites" published in Feb 2015.
- **ChaCha20** is a modern stream cipher with good performance in s/w.
  - It has been adopted as a replacement for RC4 in several internet standards.

RBG & PRBG

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Definition

*A **pseudo-random bit generator (PRBG)** is a deterministic algorithm which, given a truly random binary sequence of length k, outputs a binary sequence of length ℓ much larger than k which "appears" to be random. The input to the PRBG is called **seed**, while the output of the PRBG is called a **pseudo-random bit sequence**.*

PRBG & CSPRBG

Definition

We say that a ***PRBG passes all poly-time statistical tests*** *if no poly-time algorithm can correctly distinguish between an output sequence of the generator and a TRBG of the same length with prob significantly* $> \frac{1}{2}$.

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Definition

We say that a ***PRBG passes the next-bit test*** *if there is no poly-time algo which, on input of the first* ℓ *bits of an output sequence* s, *can predict the* $(\ell + 1)^{th}$ *bit of* s *with prob significantly* $> \frac{1}{2}$.

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Definition

A PRBG that passes the next-bit test is called a ***cryptographically secure PRBG****.*

Linear Congruential Generator

- Designed by D. H. Lehmer in 1949
- $x_n \equiv a.x_{n-1} + b \mod m$, where $n \geq 1$.
- Ouput depends on the **initial seed** x_0 and $a, b, \ \& \ m$.

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Theorem

If $b \neq 0$, LCG generates a sequence of length m iff

(i) $\gcd(b, m) = 1$,

(ii) *if $p \mid m$, then $p \mid (a-1)$ for all prime factor p of m,*

(iii) *if $4 \mid m$, then $4 \mid (a-1)$.*

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LCGs are not very useful for cryptographic purpose.

RSA CSPRBG

- Choose 2 large primes p & q.
- Set $n = p.q$
- Choose a random e s/t $0 < e < \phi(n)$ s/t $\gcd(e, \phi(n)) = 1$.
- Choose a random seed x_0 s/t $1 \leq x_0 \leq n-1$

$$x_i \equiv x_{i-1}^e \mod n.$$

- Let b_i be the least significant bit of x_i.
- ℓ random bits are $b_1, b_2, \ldots, b_\ell$.

BBS (Blum-Blum-Shub) CSPRBG

- Generate 2 large primes p & q s/t both $\equiv 3 \mod 4$
- Set $n = p.q$
- Select a random integer x s/t $\gcd(x, n) = 1$
- Set initial seed $x_0 \equiv x^2 \mod n$

$$x_i \equiv x_{i-1}^2 \mod n$$

- Let b_i be the least significant bit of x_i.
- ℓ random bits are $b_1, b_2, \ldots, b_\ell$.

Outline

Golomb's Postulates

- Let $s = s_0, s_1, s_2, \ldots$ be an infinite sequence. The subsequence consisting of the first n terms of s is denoted by $s^n = s_0, s_1, \ldots, s_{n-1}$.

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Definition

Let $s = s_0, s_1, s_2, \ldots$ be a periodic sequence of period N. The autocorrelation function of s is the integer-valued function $C(t)$ defined as

$$C(t) = \frac{1}{N}\sum_{i=0}^{N-1}(2.s_i - 1).(2s_{i+t} - 1), \quad for\ 0 \le t \le N-1.$$

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$C(t)$ measures the amount of similarity between the sequence s and a shift of s by t positions. If s is a random periodic sequence of period N, then $|N.C(t)|$ can be expected to be quite small for all values of t, $0 < t < N$.

Golomb's Postulates

Let s be a periodic sequence of period N. **Golomb's randomness postulates** are the following:

(i) In the cycle s^N of s, the number of 1's differs from the number of 0's by at most 1.

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(ii) In the cycle s^N, at least half the runs have length 1, at least one-fourth have length 2, at least one-eighth have length 3, etc., as long as the number of runs so indicated exceeds 1. Moreover, for each of these lengths, there are (almost) equally many gaps and blocks.

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(iii) The autocorrelation function $C(t)$ is two-valued. That is for some integer K,

$$N \times C(t) = \sum_{i=0}^{N-1} (2.s_i - 1).(2s_{i+t} - 1) = \begin{cases} N, & if\ t = 0, \\ K, & if\ 1 \le t \le N-1. \end{cases}$$

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A binary sequence which satisfies Golomb's randomness postulates is called a pseudo-noise sequence or a pn-sequence.

Frequency Test (Monobit Test)

- The purpose of this test is to determine whether the number of 0's and 1's in s are approximately the same, as would be expected for a random sequence.
- Let $s = s_0, s_1, s_2, \ldots, s_{n-1}$ be a binary sequence of length n.
- Let n_0, n_1 denote the number of 0's and 1's in s, respectively.
- The statistic used is

$$X_1 = \frac{(n_0 - n_1)^2}{n}$$

which approximately follows a χ^2 distribution with 1 degree of freedom if $n \geq 10$.

Serial Test (2-bit Test)

- The purpose of this test is to determine whether the number of occurrences of 00, 01, 10, and 11 as subsequences of s are approximately the same, as would be expected for a random sequence.
- Let n_0, n_1 denote the number of 0's and 1's in s, respectively, and let $n_{00}, n_{01}, n_{10}, n_{11}$ denote the number of occurrences of 00, 01, 10, 11 in s, respectively[2].
- The statistic used is

$$X_2 = \frac{4}{n-1}(n_{00}^2 + n_{01}^2 + n_{10}^2 + n_{11}^2) - \frac{2}{n}(n_0^2 + n_1^2) + 1$$

which approximately follows a χ^2 distribution with 2 degrees of freedom if $n \geq 21$.

[2] $n_{00} + n_{01} + n_{10} + n_{11} = (n-1)$ since the subsequences are allowed to overlap.

Poker test

- Let m be a positive integer such that $\lfloor \frac{n}{m} \rfloor \geq 5.2^m$, and let $k = \lfloor \frac{n}{m} \rfloor$.
- Divide the sequence s into k non-overlapping parts each of length m
- Let n_i be the number of occurrences of the i^{th} type of sequence of length m, $1 \leq i \leq 2^m$.
- The poker test[3] determines whether the sequences of length m each appear approximately the same number of times in s, as would be expected for a random sequence.
- The statistic used is

$$X_3 = \frac{2^m}{k}\left(\sum_{i=1}^{2^m} n_i^2\right) - k$$

which approximately follows a χ^2 distribution with $2^m - 1$ degrees of freedom.

[3] Note that the poker test is a generalization of the frequency test: setting $m = 1$ in the poker test yields the frequency test.

Runs test

- The purpose of the runs test is to determine whether the number of runs of various lengths in the sequence s is as expected for a random sequence.
- The expected number of gaps (or blocks) of length i in a random sequence of length n is $e_i = (n - i + 3)/2^{i+2}$.
- Let k be equal to the largest integer i for which $e_i \geq 5$.
- Let B_i, G_i be the number of blocks and gaps, respectively, of length i in s for each i, $1 \leq i \leq k$.
- The statistic used is

$$X_4 = \sum_{i}^{k} \frac{(B_i - e_i)^2}{e_i} + \sum_{i}^{k} \frac{(G_i - e_i)^2}{e_i}$$

which approximately follows a χ^2 distribution with $2k - 2$ degrees of freedom.

Autocorrelation test

- The purpose of this test is to check for correlations between the sequence s and (non-cyclic) shifted versions of it.
- Let d be a fixed integer, $1 \leq d \leq \lfloor n/2 \rfloor$.
- The number of bits in s not equal to their d-shifts is $A(d) = \sum_{i=0}^{n-d-1} s_i \oplus s_{i+d}$.
- The statistic used is

$$X_5 = \frac{2(A(d) - \frac{n-d}{2})}{\sqrt{n-d}}$$

which approximately follows an $N(0,1)$ distribution if $n - d \geq 10$. Since small values of $A(d)$ are as unexpected as large values of $A(d)$, a two-sided test should be used.

Outline

1. Introduction
2. Statistical Tests
   - Five Basic Tests
3. LFSR
4. RC4
5. Trivium
6. Salsa20/20

Linear Feedback Shift Registers (LFSR)

- A standard way of producing a binary stream of data is to use a feedback shift register.
- These are small circuits containing a number of memory cells, each of which holds one bit of information.
- The set of such cells forms a register.
- In each cycle a certain predefined set of cells are '**tapped**' and their value is passed through a function, called the feedback function.
- The register is then shifted down by one bit, with the output bit of the feedback shift register being the bit that is shifted out of the register.
- The combination of the tapped bits is then fed into the empty cell at the top of the register.

Linear Feedback Shift Registers (LFSR)

Definition

A LFSR of degree L (or length L) is defined by feedback coefficients $c_1, \ldots, c_L \in GF(2)$.

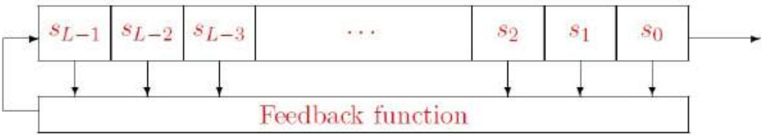
The initial state is an L-bit word $S = (s_{L-1}, \ldots, s_1, s_0)$ and new bits are generated by the recursion

$$s_j = (c_1.s_{j-1} \oplus c_2 s_{j-2} \oplus \ldots \oplus c_L.s_{j-L}) \mod 2, \; for \; j \geq L$$

At each iteration step (clock tick), the state S is updated from $(s_{j-1}, \ldots, s_{j-L})$ to $(s_j, s_{j-1}, \ldots, s_{j-L+1})$, i.e., by shifting the register to the right. The rightmost bit s_{j-L} is output.

*The output of an LFSR is called a **linear recurring sequence**.*

Linear Feedback Shift Registers (LFSR)

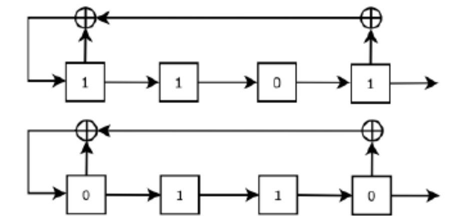


- Let the length of the register be L.
- One defines a set of bits $(c_1, \ldots, c_L)$ where $c_i = 1$ if that cell is tapped and $c_i = 0$ otherwise.
- The initial internal state of the register is given by the bit sequence $(s_{L-1}, \ldots, s_1, s_0)$.
- The output sequence is then defined to be $s_0, s_1, s_2, \ldots, s_{L-1}, s_L, s_{L+1}, \ldots$ where for $j \geq L$ we have

$$s_j = c_1.s_{j-1} \oplus c_2 s_{j-2} \oplus \ldots \oplus c_L.s_{j-L}.$$

Linear Feedback Shift Registers (LFSR)

Example (LFSR)



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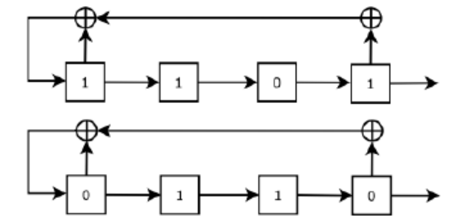
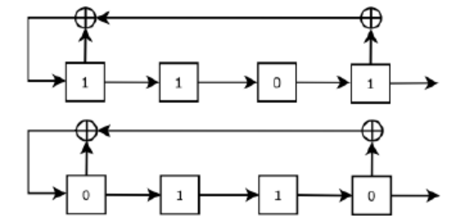


- Connection polynomial:

Linear Feedback Shift Registers (LFSR)

Example (LFSR)



- Connection polynomial: $c(x) = x^4 + x + 1$
- Initial state is $(1, 1, 0, 1)$

Linear Feedback Shift Registers (LFSR)

Example (LFSR)

$$
\begin{array}{ccc}
1101 & \rightarrow & 1 \\
0110 & \rightarrow & 0 \\
0011 & \rightarrow & 1 \\
1001 & \rightarrow & 1 \\
0100 & \rightarrow & 0 \\
0010 & \rightarrow & 0 \\
0001 & \rightarrow & 1 \\
1000 & \rightarrow & 0 \\
1100 & \rightarrow & 0 \\
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1101 & &
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$$

Linear Feedback Shift Registers (LFSR)

Definition

Let $s_0, s_1, s_2, \ldots$ *be a linear recurring sequence. The period of the sequence is the smallest integer* $N \geq 1$ *s/t*

$$s_{j+N} = s_j$$

for all sufficiently large values of j.

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for all sufficiently large values of j.

Proposition

The period of a sequence generated by an LFSR of degree n is at most $2^n - 1$.

Linear Complexity

Definition

*The **linear complexity** of an infinite binary sequence s, denoted $L(s)$, is defined as follows:*

1. (i) *if s is the zero sequence $s = 0, 0, 0, \ldots,$ then $L(s) = 0$;*
2. (ii) *if no LFSR generates s, then $L(s) = \infty$;*
3. (iii) *otherwise, $L(s)$ is the length of the shortest LFSR that generates s.*

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Definition

The linear complexity of a finite binary sequence s^n, denoted $L(s^n)$, is the length of the shortest LFSR that generates a sequence having s^n as its first n terms.

Properties of Linear Complexity

(i) For any $n \geq 1$, the linear complexity of the subsequence s^n satisfies $0 \leq L(s^n) \leq n$.

(ii) $L(s^n) = 0$ iff s^n is the zero sequence of length n.

(iii) $L(s^n) = n$ iff $s^n = 0, 0, 0, \ldots, 0, 1$.

(iv) If s is periodic with period N, then $L(s) \leq N$.

(v) $L(s \oplus t) \leq L(s) + L(t)$, where $s \oplus t$ denotes the bitwise XOR of s and t.

Non-linear FSR (NLFSR)

Example

- Consider a 4-stage NFSR with a feedback function

$$f(x_0, x_1, x_2, x_3) = 1 + x_0 + x_1 + x_1x_2x_3$$

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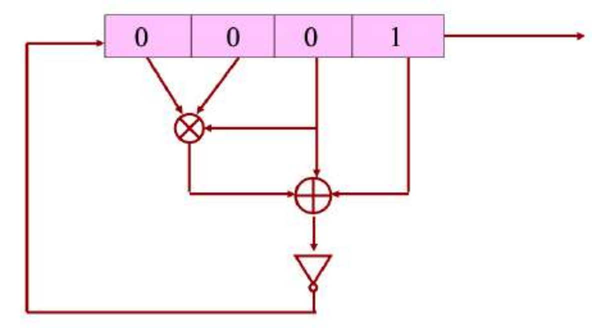

Non-linear FSR (NLFSR)

Example

$f(x_0, x_1, x_2, x_3) = 1 + x_0 + x_1 + x_1 x_2 x_3$ – de Bruijn FSR

$$
\begin{array}{ccc}
0001 & \rightarrow & 1 \\
0000 & \rightarrow & 0 \\
1000 & \rightarrow & 0 \\
1100 & \rightarrow & 0 \\
1110 & \rightarrow & 0 \\
1111 & \rightarrow & 1 \\
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1011 & \rightarrow & 1 \\
1101 & \rightarrow & 1 \\
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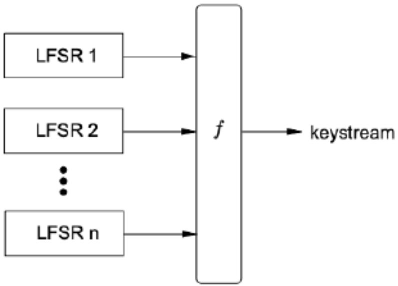
0011 & \rightarrow & 1 \\
1001 & \rightarrow & 1 \\
0100 & \rightarrow & 0 \\
1010 & \rightarrow & 0 \\
0101 & \rightarrow & 1 \\
0010 & \rightarrow & 1
\end{array}
$$

Stream Ciphers Based on LFSRs

- **combination generator**
- **filter generator**
- **shrinking generator**

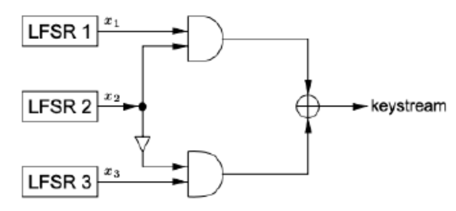
Non-linear Combination Generator

- One general technique for destroying the linearity inherent in LFSRs is to use several LFSRs in parallel.
- The key stream is generated as a non-linear function f of the outputs of the component LFSRs.
- Such key stream generators are called non-linear combination generators, and f is called the combining function.



Non-linear Combination Generator

Example (Geffe Generator)



- Consider 3 maximum-length LFSRs whose lengths L_1, L_2, L_3 are pairwise relatively prime, with nonlinear combining function

$$f(x_1, x_2, x_3) = x_1x_2 \oplus (1 + x_2)x_3 = x_1x_2 \oplus x_2x_3 \oplus x_3.$$

Non-linear Combination Generator

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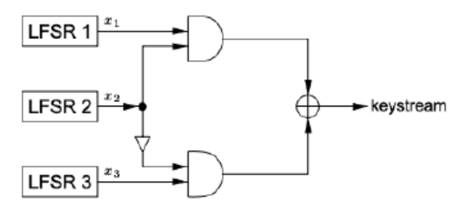

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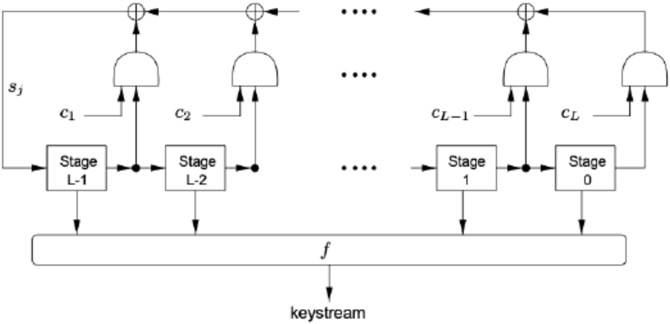
$$f(x_1, x_2, x_3) = x_1x_2 \oplus (1 + x_2)x_3 = x_1x_2 \oplus x_2x_3 \oplus x_3.$$

- The keystream generated has period $(2^{L_1} - 1)(2^{L_2} - 1)(2^{L_3} - 1)$ and linear complexity $L = L_1L_2 + L_2L_3 + L_3$.

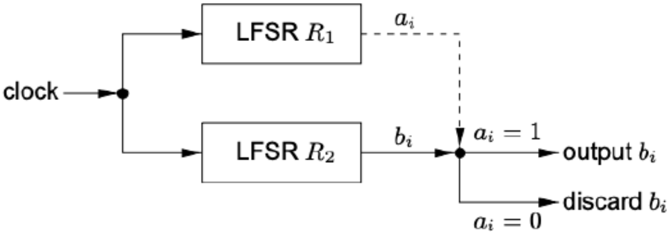
Filter Generator

- A filter generator is a running-key generator for stream cipher applications.
- It consists of a single LFSR which is filtered by a non-linear function f.



Shrinking Generator

- A control LFSR R_1 is used to select a portion of the output sequence of a second LFSR R_2
- Due to its simplicity, it was a promising candidate for high-speed encryption applications.



Outline

RC4

- A self-modifying lookup table (or Synchronous stream cipher) designed by Ron Rivest in 1987.
- Table always contains a permutation of the byte values $0, 1, \ldots, 255$
- Initialize the permutation using key
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- Each step of $A5/1$ produces only a bit
  - Efficient in hardware

RC4 Key Scheduling Algorithm (KSA)

Input: Key array $K[0], K[1], \ldots, K[n-1]$ of n bytes, $1 \leq n \leq 255$
Output: State array $S[0], S[1], \ldots, S[255]$

for $i = 0$ to 255 **do**
    $S[i] = i$
end for
$j = 0$
for $i = 0$ to 255 **do**
    $j = (j + S[i] + K[i \bmod n]) \bmod 256$
    Swap the values of $S[i]$ and $S[j]$
end for

RC4 Pseudorandom Generation Algorithm (PRGA)

- For each keystream byte, swap elements in table and select byte

Input: State array $S[0], S[1], \dots, S[255]$
Output: Output bytes B

$i = 0$
$j = 0$
while Keystream is generated **do**
    $i = i + 1$
    $j = (j + S[i]) \bmod 256$
    Swap the values of $S[i]$ and $S[j]$
    $B = S[(S[i] + S[j]) \bmod 256]$
    Output B
end while

- Use keystream bytes like a one-time pad
- Note: first 256 bytes should be discarded
  - Otherwise, related key attack exists

Outline

Trivium

- Designed by De Canniére and Preneel in 2006 as part of eSTREAM competition
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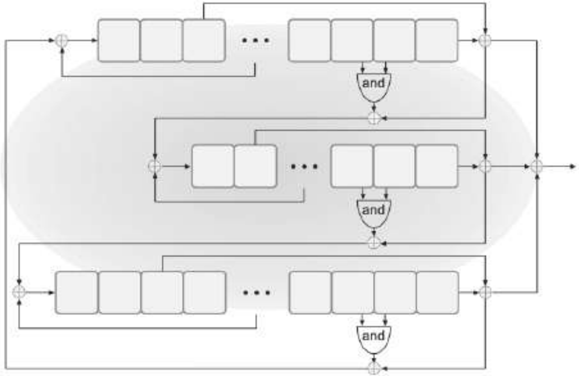
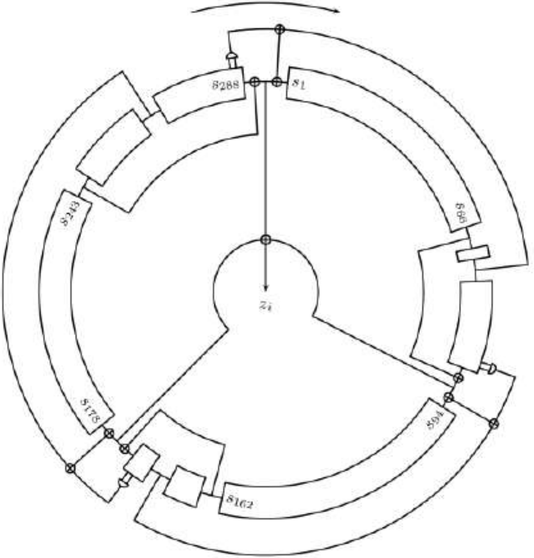


Trivium Hardware



Trivium

- **Parameters:**
  Key size: 80 bit, IV size: 80 bit, Internal state: 288 bit

Trivium

- **Parameters:**
  Key size: 80 bit, IV size: 80 bit, Internal state: 288 bit
- Three coupled FSR of degree 93, 84, and 111.
- **Initialization:**
  - 80-bit key in left-most registers of first FSR
  - 80-bit IV in left-most registers of second FSR
  - Remaining registers set to 0, except for three right-most (all 1s) registers of third FSR
  - run for 4×288 clock ticks to finish initialization

https://www.ecrypt.eu.org/stream/p3ciphers/trivium/trivium_p3.pdf

Trivium-Initialization

For $i = 1$ to 4×288 do

1. $t_1 \leftarrow s_{66} + s_{91}s_{92} + s_{93} + s_{171}$
2. $t_2 \leftarrow s_{162} + s_{175}s_{176} + s_{177} + s_{264}$
3. $t_3 \leftarrow s_{243} + s_{286}s_{287} + s_{288} + s_{69}$
4. $(s_1, s_2, \ldots, s_{93}) \leftarrow (t_3, s_1, \ldots, s_{92})$
5. $(s_{94}, s_{95}, \ldots, s_{177}) \leftarrow (t_1, s_{94}, ..., s_{176})$
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Note: no random bits output. This is just initialization.

Trivium-Iteration

For $i = 1$ to $N(\leq 2^{64})$ do

1. $t_1 \leftarrow s_{66} + s_{93}$
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4. $z_i \leftarrow t_1 + t_2 + t_3$

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4. $z_i \leftarrow t_1 + t_2 + t_3$ **1 bit of key stream**
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Outline

Salsa20/20

- Salsa20 is based on three simple operations:
  - modular addition of 32-bit words a and $b \mod 2^{32}$, denoted by $a \boxplus b$,
  - XOR-sum of 32-bit words a and b, denoted by $a \oplus b$,
  - circular left shift of a 32-bit word a by t positions, denoted by $a \lll t$.

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- The Salsa20/20 cipher takes a 256-bit key, a 64-bit nonce and a 64-bit counter.
- The state array S of Salsa20 is a 4×4 matrix of sixteen 32-bit words[4]

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Salsa20/20

The state array S:

$$S = \begin{pmatrix} y_0 & y_1 & y_2 & y_3 \\ y_4 & y_5 & y_6 & y_7 \\ y_8 & y_9 & y_{10} & y_{11} \\ y_{12} & y_{13} & y_{14} & y_{15} \end{pmatrix}$$

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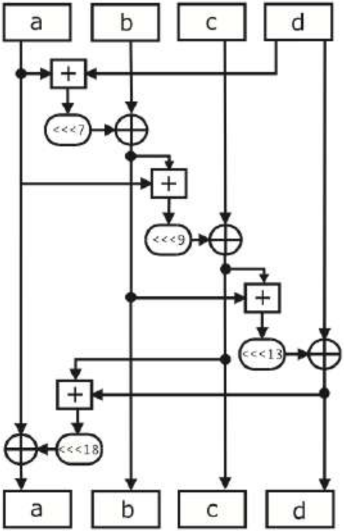
- Salsa20 is based on **quarter-rounds**, **row-rounds** and **column-rounds**.
- The quarter-rounds operate on four words, the row-rounds transform the four rows and the column-rounds transform the four columns of the state matrix.

Salsa20/20: Quarter-round



Salsa20/20: Row-round

$$row - round(S) = \begin{pmatrix} z_0 & z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 & z_7 \\ z_8 & z_9 & z_{10} & z_{11} \\ z_{12} & z_{13} & z_{14} & z_{15} \end{pmatrix},$$

where

$$(z_0, z_1, z_2, z_3) = \text{quarter-round}(y_0, y_1, y_2, y_3),$$

$$(z_5, z_6, z_7, z_4) = \text{quarter-round}(y_5, y_6, y_7, y_4),$$

$$(z_{10}, z_{11}, z_8, z_9) = \text{quarter-round}(y_{10}, y_{11}, y_8, y_9),$$

$$(z_{15}, z_{12}, z_{13}, z_{14}) = \text{quarter-round}(y_{15}, y_{12}, y_{13}, y_{14}).$$

Salsa20/20: Column-round

- The *column-round* function is the transpose of the row-round function: the words in the columns are permuted, the quarter-round map is applied to each of the columns and the permutation is reversed.
- Let S be a state matrix as above; then

$$\text{column-round}(S) = (\text{row-round}(S^T))^T.$$

Salsa20/20: Double-round

- A *double-round* is the composition of a column-round and a row-round.
- Let S be a state matrix as above; then

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Salsa20 runs 10 successive double-rounds, i.e., 20 quarter-rounds, in order to generate 64 bytes of output.

The *initial state* depends on the *key*, a *nonce* and a *counter*.

Salsa20/20

- The Salsa20/20 stream cipher takes a 256-bit key $k = (k_1, \ldots, k_8)$ and a unique 64-bit message number $n = (n_1, n_2)$ (nonce) as input.
- A 64-bit block counter $b = (b_1, b_2)$ is initially set to zero.
- The initialization algorithm copies k, n, b and the four 32-bit constants

$$y_0 = 61707865,\ y_5 = 3320646E,\ y_{10} = 79622D32,\ \&\ y_{15} = 6B206574$$

into the sixteen 32-bit words of the Salsa20 state matrix:

Salsa20/20

The state array S:

$$S = \begin{pmatrix} y_0 & k_1 & k_2 & k_3 \\ k_4 & y_5 & n_1 & n_2 \\ b_1 & b_2 & y_{10} & k_5 \\ k_6 & k_7 & k_8 & y_{15} \end{pmatrix}$$

- The key stream generator computes the output state by *10 double-round* iterations and a final addition $\mod 2^{32}$ of the initial state matrix:

$$Salsa20_k(n, b) = S + \text{double-round}^{10}(S).$$

Salsa20/20

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ChaCha20 is a modification of Salsa20

ChaCha20

- ChaCha20 is a stream cipher intended to be extremely efficient in s/w, introduced in 2008.
- It is available as a replacement for RC4 in many systems.
- It is combined with the Poly1305 message authentication code to construct an authenticated encryption (AE) scheme widely used in the TLS protocol.

ChaCha20 Quarter-round

- Let $y = (a, b, c, d)$ be a sequence of four 32-bit words.
- Then a ChaCha quarter-round updates (a, b, c, d) as follows:

  (i) $a = a + b; \quad d = d \oplus a; \quad d \lll 16;$

  (ii) $c = c + d; \quad b = b \oplus c; \quad b \lll 12;$

  (iii) $a = a + b; \quad d = d \oplus a; \quad d \lll 8;$

  (iv) $c = c + d; \quad b = b \oplus c; \quad b \lll 7;$

CChaCha20 Double-round

- ChaCha20 also runs 10 double-rounds.
- However, a ChaCha double-round consists of a column-round and a diagonal-round
- A ChaCha double-round is defined by the 8 ChaCha quarter-rounds

column-round	quarter-round(y_0, y_4, y_8, y_{12}) quarter-round(y_1, y_5, y_9, y_{13}) quarter-round$(y_2, y_6, y_{10}, y_{14})$) quarter-round$(y_3, y_7, y_{11}, y_{15})$
diagonal-round	quarter-round$(y_0, y_5, y_{10}, y_{15})$ quarter-round$(y_1, y_6, y_{11}, y_{12})$ quarter-round(y_2, y_7, y_8, y_{13}) quarter-round(y_3, y_4, y_9, y_{14})

ChaCha20

The state array S:

$$S = \begin{pmatrix} y_0 & y_1 & y_2 & y_3 \\ k_1 & k_2 & k_3 & k_4 \\ k_5 & k_6 & k_7 & k_8 \\ b & n_1 & n_2 & n_3 \end{pmatrix}$$

- The ChaCha20 stream cipher takes a 256-bit key $k = (k_1, \ldots, k_8)$ and a unique 96-bit message number $n = (n_1, n_2, n_3)$ (nonce) as input.
- A 32-bit block counter b is initially set to zero and the four 32-bit constants

$$y_0 = 61707865, y_1 = 3320646E, y_2 = 79622D32, y_3 = 6B206574$$

$$ChaCha_k(n, b) = S + \text{double-round}^{10}(S).$$

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  - Today, processors are fast, so software-based crypto is usually more than fast enough
- Future of stream ciphers?
  - Shamir declared "the death of stream ciphers"
  - May be greatly exaggerated ...

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The End

Thank you very much for your attention!